It may be noted here that there are 6 (4+3-1) allocations which are necessary to proceed further.

Assignment Problems

Q.41. Discuss practical application of an assignment problem. Solve the following assignment problem.

Worker	Job-1	Job-2	Job-3
A	4	2	7
B	8	5	3
C	4	5	6

Time is stated in minutes.

Solution :

Step I : Select the smallest element in each row and subtract it from every element of that row.

Here, 2,3,4 are respectively the smallest elements in A, B,C rows.

A	2	0	5
B	5	2	0
C	0	1	2

Step II: Subtract the smallest element in each column from every element of that column. We see that there is zero (smallest element) in each and every column. Hence, Column reduced matrix will be the

	1	2	3
A	2	[0]	5
B	5	2	[0]
C	[0]	1	2

same as above given Row Reduced Matrix.

Step III : Assignment Matrix

Optimal Assignment A→2, B→3 and C→1

Total Minimum Time = 2 +3 + 4 = 09 minutes.

Q.42. Find the optimal solution for the assignment problem with the following cost matrix :

Salesman	Area			
	W	X	Y	Z
A	11	17	8	16
B	9	7	12	6
C	13	16	15	12
D	14	10	12	11

Solution :

To find out the optimal solution, step-by-step procedure is as follows :

Step 1 : Row Reduction

Select the smallest element in each row and subtract it from every element of that row. Here 8, 6, 12 and 10 are the smallest elements in rows A, B, C and D respectively. Subtracting these elements from every row, we get the reduced matrix as given below in table 1.

Table 1

Salesman \ Area	W	X	Y	Z
A	3	9	0	8
B	3	1	6	0
C	1	4	3	0
D	4	0	2	1

Step 2 : Column Reduction

Select the smallest element in each column of above reduced matrix and subtract it from every element of that column. Now in the reduced matrix, columns W, X, Y and Z have 1, 0, 0 and 0 as

परिचालन शोध

Table 2

Area Salesman	W	X	Y	Z
A	2	9	0	8
B	2	1	6	0
C	0	4	3	0
D	3	0	2	1

Step 3 : Making Assignments

Now we make the assignments as follows :

(a) Starting with row 'A' of the reduced matrix given in table 2, we examine all the rows one by one until a row containing exactly single zero elements is found. Here, row A, B and D are the rows which have single zero. So, make the assignment to the respective zero element by putting and strike off (×) other zeros in the column in which the assignment is made. This is shown in table 3.

Table 3

Area Salesman	W	X	Y	Z
A	2	9	[0]	8
B	2	1	6	[0]
C	[0]	4	3	⊠
D	3	[0]	2	1

(b) When rows have been examined successively, an identical procedure is applied to each and every column. Starting with column W, we examine all the columns until a column containing exactly one remaining zero is found. Since the number of assignments □ made are equal to the number of rows/column (i.e. 4), we have obtained the optimal solution. The complete set of assignments of areas to salesman are given below :

Salesman	Area	Cost (₹)
A	Y	8
B	Z	6
C	W	13
D	X	10
		M.T.C. 37